Exercise 1: Basic programming in R

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Part 1: Matrix and vector operations.

1. Solve the following system:

$$\begin{bmatrix} a_1 & b_1 & & & 0 \\ c_1 & a_2 & b_2 & & \\ & \ddots & \ddots & \ddots & \\ & & & a_{99} & b_{99} \\ 0 & & & c_{99} & a_{100} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{100} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{100} \end{bmatrix}$$

where

$$a_j = j, \quad b_j = 1/j, \quad c_j = 1, \quad d_j = \sin(j\pi/200)$$

and print $x_1, x_2, \ldots, x_5$.

Part 2: For loops.

1. Write a function that uses a `for` loop to calculate the following with a sequence of m, and generate a plot for m verses E_m. Avoid using a `for` loop, can you complete the same task?

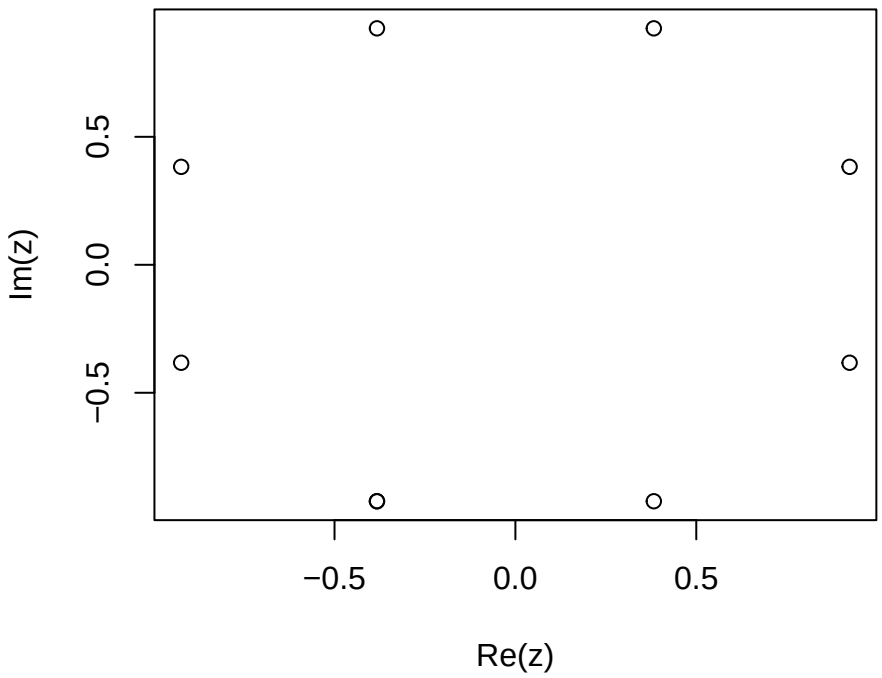
$$E_m = 1 + \frac{1}{2} + \cdots + \frac{1}{2^m} - \log(2^m)$$

2. Let's draw a regular polygon of n sides, with a horizontal bottom side, and the corners of the polygon staying on the unit circle. For given $n = 5$ and $r = 0.6$, start the first point $p_1 = (x_1, y_1)$ as a random number in $(-0.5, 0.5) \times (-0.5, 0.5)$, generate 10^4 points interactively. In the jth iteration, we choose one corner z_* of the polygon randomly and let $p_{j+1} = (x_{j+1}, y_{j+1})$ be the point on the line segment between p_j and z_*, with the distance from p_{j+1} to p_j being r times the distance from z_j to p_j and then draw all these points as dots in the xy panel.

Hint: Complex numbers can be used to represent points in the xy plane. The following script works for even or odd n, and the polygon always has a flat bottom.

```
n <- 8
t <- c(0:n) - 0.5
z <-  exp(2i * pi * (t/n - 0.25))

plot(Re(z), Im(z))
```


Im(z)
0.5
0.0
−0.5
−0.5
0.0
0.5
Re(z)