

Exercise 4: Statistical inference (I)

July 16, 2024

Part 1: Probability distributions

1. A contestant on a game show needs to answer 10 questions correctly to win the jackpot. However, if they get 4 incorrect answers, they are kicked off the show. Suppose one contestant consistently has a 80% chance of correctly responding to any question.
 - (a) What is the probability distribution?
 - (b) What is the probability that she will correctly answer 10 questions before 4 incorrect responses?
 - (c) Write out the R code to calculate (b).
2. A small town's police department issues 5 speeding tickets per month on average.
 - (a) Using a Poisson random variable, what is the likelihood that the police department issues 3 or fewer tickets in one month?
 - (b) What is the probability that 10 days or fewer elapse between two tickets being issued?
 - (c) Write out the R code to calculate (a), (b).

Part 2: Statistical inference

1. (AoS 6.6.2) Let $X_1, \dots, X_n \sim \text{Uniform}(0, \theta)$ and let $\hat{\theta} = \max\{X_1, \dots, X_n\}$. Find the bias, se and MSE of this estimator.
2. (AoS 6.6.3) Let $X_1, \dots, X_n \sim \text{Uniform}(0, \theta)$ and let $\hat{\theta} = 2\bar{X}_n$. Find the bias, se and MSE of this estimator.
3. Let $X_1, \dots, X_n \sim \text{Uniform}(0, 1)$. Let $Y_n = \bar{X}_n^2$. Find the limiting distribution of Y_n . (Hint: CLT)