Exercises for Module 9: Linear Algebra III

1. Let $A, B \in M_n(\mathbb{F})$ be similar matrices. Show that their characteristic polynomials coincide.

2. Show that $A \in M_n(\mathbb{C})$ is invertible if and only if $0 \notin \sigma(A)$.

3. Suppose N is a nilpotent matrix. Show that $\sigma(N) = \{0\}$.

4. Let $A \in M_n(\mathbb{C})$ be an invertible matrix. Show that λ is an eigenvalue of A if and only if λ^{-1} is an eigenvalue of A^{-1}.

5. Suppose $A \in M_n(\mathbb{C})$ is Hermitian. Show that all the eigenvalues of A are real. Hint: Note that if $\mathbf{x}$ is a normalized eigenvector of A with eigenvalue λ, then $\langle A\mathbf{x}, \mathbf{x} \rangle = \lambda$.

6. Let $A \in M_n(\mathbb{R})$. Show that the eigenvalues of $A^T A$ are non-negative.