

## Exercises for Module 8: Linear Algebra II

1. Let  $D \in \mathcal{L}(\mathbb{P}_4(\mathbb{R}), \mathbb{P}_3(\mathbb{R}))$  be the differentiation map,  $Dp = p'$ . Find bases of  $\mathbb{P}_4(\mathbb{R})$  and  $\mathbb{P}_3(\mathbb{R})$  such that the matrix representation of  $\mathcal{M}(D)$  with respect to these basis is given by

$$\mathcal{M}(D) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

2. Show that matrix multiplication of square matrices is not commutative, i.e find matrices  $A, B \in M_2$  such that  $AB \neq BA$ .

3. A square matrix is called *nilpotent* if  $\exists k \in \mathbb{N}$  such that  $A^k = 0$ . Show that for a nilpotent matrix  $A$ ,  $|A| = 0$ .

4. A real square matrix  $Q$  is called *orthogonal* if  $Q^T Q = I$ . Prove that if  $Q$  is orthogonal, then  $|Q| = \pm 1$ .

5. An  $n \times n$  matrix is called *antisymmetric* if  $A^T = -A$ . Prove that if  $A$  is antisymmetric and  $n$  is odd, then  $|A| = 0$ .

6. Let  $V$  be an inner product space,  $U$  a vector space and  $S: U \rightarrow V$ ,  $T: U \rightarrow V$  be linear maps. Show that  $\langle S\mathbf{u}, \mathbf{v} \rangle = \langle T\mathbf{u}, \mathbf{v} \rangle$  for all  $\mathbf{u} \in U$  and  $\mathbf{v} \in V$  implies  $S = T$ .