UNIVERSITY OF TORONTO | Statistical Sciences

DoSS Summer Bootcamp Probability Module 8

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Recap

Learnt in last module:

- Stochastic convergence
  - ▷ Convergence in distribution
  - ▷ Convergence in probability
  - ▷ Convergence almost surely
  - ▷ Convergence in L^p
  - ▷ Relationship between convergences

Outline

- Relationship between convergences and counterexamples
  - Monotonicity of L^p Convergence
  - L^p convergence implies Convergence in Probability
  - a.s. Convergence implies Convergence in Probability
  - Convergence in Probability implies Convergence in distribution

Relationship between convergences and counterexamples

Recall: A random variable $X \in L^p$ if $\|X\|_{L^p} = (E|X|^p)^{1/p} < \infty$.

$X_n \to X$ in L^p if $\lim_{n\to\infty} \|X_n - X\|_{L^p} = 0$

Monotonicity of L^p Convergence

If $q > p > 0$, L^q convergence implies L^p convergence

Proof:

Relationship between convergences and counterexamples

Recall: A random variable $X \in L^p$ if $\|X\|_{L^p} = (E|X|^p)^{1/p} < \infty$.

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Monotonicity of L^p Convergence

If $q > p > 0$, L^q convergence implies L^p convergence

Counterexample to the Converse:

Relationship between convergences and counterexamples

Recall: X_n converges to X in probability if for any $\epsilon > 0$ $\lim_{n\to\infty} P(|X_n - X| > \epsilon) = 0$.

L^p convergence implies Convergence in Probability

If $X_n \to X$ in L^p, then $X_n \to X$ in probability.

Proof:

Relationship between convergences and counterexamples

Recall: X_n converges to X in probability if for any $\epsilon > 0$ $\lim_{n\to\infty} P(|X_n - X| > \epsilon) = 0$.

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a.s. Convergence implies Convergence in Probability

If $X_n \to X$ almost surely, then $X_n \to X$ in probability.

Proof:

Relationship between convergences and counterexamples

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a.s. Convergence implies Convergence in Probability

If $X_n \to X$ almost surely, then $X_n \to X$ in probability.

Counterexample to the Converse:

Relationship between convergences and counterexamples

Recall: X_n converges to X in distribution if for any continuity point x of $P(X \leq x)$, $\lim_{n\to\infty} P(X_n \leq x) = P(X \leq x)$ holds.

Convergence in Probability implies Convergence in Distribution

If $X_n \to X$ in probability, then $X_n \to X$ in distribution.

Proof:

Relationship between convergences and counterexamples

Recall: X_n converges to X in distribution if for any continuity point x of $P(X \leq x)$, $\lim_{n\to\infty} P(X_n \leq x) = P(X \leq x)$ holds.

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Convergence in Probability implies Convergence in Distribution

If $X_n \to X$ in probability, then $X_n \to X$ in distribution.

Special case when the Converse holds: